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MATHEMATICS CONTESTS.*

By Ernest H. Koch, Jr.

From time immemorial games have been a necessary complement to the more austere forms of intellectual and social development. We have historic records of contests which enable us to obtain a familiar acquaintance with the manners and customs of nations as well as their achievements. To some of us these ancient games were only exhibitions of physical prowess while to others the staging of the orators and actors was a surpassing achievement. In our day we see these contest ideas culminating in great world expositions, art exhibitions, commercial shows and last but not least in the community masques. Cities vie with cities in resetting the contest motive so that each successful affair surpasses its predecessor in beauty and magnificence. International world fairs have permeated the nations, states, municipalities, colleges and schools with the spirit of the contest.

It is our purpose to show how the idea of a school contest has taken an intellectual form although modeled and staged after the physical counterparts on the track and field.

> "What dire offence from amorous causes springs,
> What mighty contests rise from trivial things!"

In the class room we may set apart a definite period during which the pupils may participate in various games, such as

* Presented at the Baltimore meeting of the Association, December 2, 1916.

checkers, chess or any of the many so-called children's games. These are intended more for the individual than for the group. All games are events of skill in the manipulation of fixed pieces, cards, devices or even individuals in which the element of distribution or motion is only partially under mental control owing to the limitations imposed by the rules of the game. A scholastic contest is a matching of intellects in a set task in which the written or verbal arts of expression play a minor rôle. All contests have their stimulus in the award of medals, banners and the publicity of honors. Any school subject may be used to afford material for a contest. In a commercial school the commercial subjects should have the greatest prominence. A Three "R" Contest is a competitive exhibition in the fundamentals of commercial education. This unique form of school activity was instituted for the purpose of interesting the public and the parents in the work of the school and for promoting the interest of the pupils in those studies which form the foundation of their life work. A program of the events of such a contest follows:

1. Addition
2. Subtraction
3. Multiplication
4. Division
5. Relay—four fundamental operations combined
6. Scissors—a crisscross in addition
7. Commercial geography
8. Spelling
9. Current events
10. Penmanship
11. Typewriting
12. Stenography

Such a program will give entertainment for two hours and offers the best reply and refutation to that part of the public which so glibly accuses the public schools of graduating poorer material than was graduated years ago. It should be observed that the mathematics takes up half of the program. These programs are altered by substitution of other subjects in the course, such as algebra, geometry, interest, bookkeeping, letter-writing, Spanish and public speaking. Contests take various forms, depending upon the method of public exhibition and demonstra-

tion and also upon the special requirements in the preliminary contests of elimination.

In the March, 1916, number of the MATHEMATICS TEACHER and June, 1916, number of *School Science and Mathematics* Mr. John H. McCormick and I have presented an account of a mathematics relay which has been tried in many secondary and elementary schools. The speaker who follows me will illustrate another form of relay which was originated and introduced independently in the William Penn High School of Philadelphia. The latter has been applied to both arithmetic and algebra. One year ago I learned of another independent contribution which was introduced in the Chicago schools. It is evident that these spontaneous and independent efforts in mathematical contests obey the same law of evolution which has given simultaneous and independent birth to identical inventions and to other forms of contemporary expressions of scientific, social and economic thought.

> "Great contest follows, and much learned dust
> Involves the combatants; each claiming truth,
> And truth disclaiming both."

With these general introductory remarks I shall proceed to describe the mathematics relay as it has been developed at the High School of Commerce in New York City. A relay consists of four related parts presented in the following order: a continued addition, a continued subtraction, a continued multiplication and a long division. Four pupils constitute a team. Each member of the team performs and completes one of the four fundamental operations described above. This will be illustrated at the blackboard to make the situation and the performance clear. (Through the courtesy of the Eastern High School of Baltimore several teams of girls demonstrated the relay.) In the addition work two addends are written at a given signal. From these seven, eight or ten addends are formed according to the instructions. Each new addend is formed by adding the two preceding ones. All the addends are then summed and when the total is correct the subtractor proceeds and subtracts a given subtrahend five times in succession. He in turn is followed by the multiplier and the latter by the divider according to their respective assignments. The team finishing first is declared the

winner. The mechanics for handling the situation is described in the article referred to.

It is possible to get the entire class interested in the relays and in each of the four fundamental operations any one of which lends itself to the contest idea. In a very short time you will find groups of pupils proficient in one or more of the fundamentals. The entire class is divided into teams of four with a captain for each team. Every member of the class can do one of the operations well and is therefore on a team. The class competition becomes so keen that a representative team is soon chosen together with an alternate team to represent the class. These are often replaced through failure to win from a challenging team of the class. A round robin is arranged for the classes in the building. The most successful of these teams represents the building. The next step is the inter-annex contest and this leads to an all school team ready to meet all comers from other schools in the city. The final contest is an intercity contest but plans for this are yet in the making.

The relay always serves as an excellent drill in the class room and for outside assignment as a very simple change in the end number of the first addend makes all of the examples different yet very easy for quick verification on the part of the teacher. The subtrahend may be chosen so as to have a definite relation to the addends. Likewise the divisor may be chosen so as to have some definite relation to the multipliers. It has been found to be better to use nine addends or twelve addends after the ten addend work has been developed. If we designate the two addends by x and y, we have the following new addends and their subtotals. A number of ingenious combinations may be made from these algebraic relations so as to get an answer quickly from the original addends. Note the formation of coefficients.

	Addends	Subtotals
(1)	x	x
(2)	y	$x + y$
(3)	$x + y$	$2x + 2y$
(4)	$x + 2y$	$3x + 4y$
(5)	$2x + 3y$	$5x + 7y$
(6)	$3x + 5y$	$8x + 12y$

(7)	$5x + 8y$	$13x + 20y$
(8)	$8x + 13y$	$21x + 33y$
(9)	$13x + 21y$	$34x + 54y$
(10)	$21x + 34y$	$55x + 88y$
(11)	$34x + 55y$	$89x + 143y$
(12)	$55x + 89y$	$144x + 232y$
(13)	$89x + 144y$	$234x + 376y$
(14)	$144x + 233y$	$378x + 699y$

In the following examples for relay work twelve addends are required and the subtrahend is used five times in succession.

(1)
73659
85673
30483032 sum
sub. 5196783
4499117 rem.
mul. 9, 7, 6
1700666226 prod.
div. 189
8998234 quot.

(2)
98967
83958
33729504 sum
sub. 5431896
6570024 rem.
mul. 9, 8, 7
3311292096 prod.
div. 756
4380016 quot.

(3)
87657
98976
35585040 sum
sub. 5638972
7390180 rem.
mul. 9, 8, 6
3192557760 prod.
div. 864
3695090 quot.

(4)
89678
56783
26087288 sum
sub. 4658937
2792603 rem.
mul. 9, 8, 7
1407471912 prod.
div. 504
2792603 quot.

In these contests it will be found advantageous to use squared paper or square ruled blackboards.

Another highly interesting series of contests is known as the Scissors or Criss-cross. This may be applied to the fundamental operations involving integers, fractions or decimals in the same manner as the relay and can be used for horizontal as well as vertical addition and subtraction.

The Scissors Applied to Addition.

A team of two pupils is designated "A" and "B." A and B begin work simultaneously at a given signal. They receive their respective assignments on library cards and transcribe these to the blackboard and then "blaze away." A is assigned addends (1) and (2) whereas B is assigned addends (10) and (11). They add in the relay fashion forming addends by adding each addend to the preceding one until eight addends have been written. The eight addends are summed. A and B interchange places, check their partner's work and then write their sum under that of the partner. They proceed in the new place making new addends until six have been written, then these are summed. A and B again interchange places, check and add their own last total to the partner's total, observing that the grand totals agree. The two pair of initial addends are composed of different numbers as shown below:

A begins here				B begins here
(1 or x)	396	these two addends are assigned	693	(10 or w)
(2 " y)	487	" " " " "	784	(11 or z)
(3)	883		1477	(12)
(4)	1370		2261	(13)
(5)	2253		3738	(14)
(6)	3623		5999	(15)
(7)	5876		9737	(16)
(8)	9499		15736	(17)

B continues here				A continues here
(9)	24387	A & B interchange places, check	40425	(18)
(18)	40425	partner's work, transfer sums	24387	(9)
(19)	64812		64812	(24)
(20)	105237		89199	(25)
(21)	179049		154011	(26)
(22)	275286		243210	(27)
(23)	680196	A & B interchange places, check	616044	(28)

A resumes here				B resumes here
(28)	616044	partner's work, transfer sums	680196	(23)
(30)	1296240	and add	1296240	(29)

The answer always ends in a cipher and is 20 times the sum of (9) and (18). An increase of 1 in either (1) or (10) increases the answer by 420. An increase of 1 in either (2) or (11) increases the answer by 660. The final sum equals 420 $(x+w)$ $+ 660$ $(y+z)$. Therefore if x is increased by the same amount which is subtracted from w the answer remains unchanged. This is likewise true for the combination $y+z$. It is this observation that enables us to set any number of addend pairs which will produce a given result, obviating copying of work by pupils.

If the addends are chosen so that $(x+w)=1000$ and also $(y+z)=1000$ the answer reduces to 1080000 which may be used as a key by means of which a number of examples may be set without effort as shown below:

x 306	694 w	$x+w=1000$	
y 487	513 z	$y+z=1000$	sum $=1080000$

If the addends are chosen so as to form pairs of complementary numbers which are multiples of 100 another set of examples may be formed as follows:

x	396	693 w
$x+p$	404	707 $w+q$

$$\text{sum}=1080(x+w)+660(p+q)=1190640$$

If $(x+w)=1000$ then this reduces to $1080000+660$ $(p+q)$ as shown below:

x	396	604 w	$x+w=1000$
$x+p$	404	796 $w+q$	$p+q\ =8+192=200$

$$\text{sum}=1080000+660(200)=1212000$$

The following examples are appended for reference:

Examples whose sum $=1080000$:

396	604	396	604	397	693	395	605
487	513,	488	512,	489	511,	480	520,

Examples whose sum $=1080660$:

396	604	396	604	397	693	395	605
487	514,	488	513,	489	512,	480	521,

Examples whose sum = 1298400:

397	694	396	695	398	693	397	694
488	785,	487	786,	489	784,	487	786,

Examples whose sum = 1299060:

415	676	415	676	417	674	416	675
489	785,	488	786,	488	786,	487	787,

Examples whose sum = 3459060:

1415	1676	1415	1676	1417	1674	1416	1675
1489	1785,	1488	1786,	1488	1786,	1487	1787,

Examples whose sum = 345906:

141.5	167.6	141.5	167.6	141.7	167.4
148.9	178.5,	148.8	178.6,	148.8	178.6,

It may be possible to prevail upon the editors of the Mathematics Teacher to give space for the publication of the activities of an intercity mathematics club. This space could be devoted to the activities of mathematics clubs, notices of contests and results. Under the auspices of the mathematics teacher and the local school organizations an intercity contest could be conducted by telephone or by having teams visit other city teams. The pleasure attending such a visit would prove a desirable incentive for a very active participation in the tryout contests. Arrangements could be made so that the expense of such a trip would not exceed the carfare for travel. The members of visiting teams would be distributed and entertained by the teachers of that school which acts as host.

"In their games children are actors, architects, and poets, and sometimes musical composers as well."

High School of Commerce,
New York City.

RETURNS TO THE QUESTIONER.

By C. C. Grove.

There still are those who think the professor has an easy time and proceed to help make him earn his "large" salary by asking him questions, with or without a stamp for reply. It seems to be expected that he is a cistern if not a spring, but they forget at what expense a good cistern is built and a gushing spring forced.

Two types of problems have come in rather frequently so that it seems worth while to make them known as a class so that when their simplicity is perceived they need not come any more for solution.

First a simple one: What must I pay annually, beginning at the date of purchase, to pay for a property valued at V dollars and interest at the end of each year at r per cent. for unpaid amount, so that all is paid at the expiration of t years?

The method of solution is to set down the situation at the end of each year in symbolic form and from analogy write out the situation at the end of the transaction; thus,

At end of first year $(V-A)(1.0r)-A$. Amount of unpaid money less A.

At end of second year $V(1.0r)^2-A(1.0r^2+1.0r+1)$. Amount of former balance less A.

At end of third year $V(1.0r)^3-A(1.0r^3+1.0r^2+1.0r+1)$.

At end of tth year $V(1.0r)^t-A(1.0r^t+1.0r^{t-1}+\cdots+1.0r+1)=0$.

Multiplying and dividing the series after A by $(1.0r-1)$, we get

$$V(1.0r)^t - A\,\frac{(1.0r)^{t+1}-1}{1.0r-1} = 0.$$

By taking logarithms of $1.0r$, multiplying by t and $(t+1)$, and finding the antilogarithms of the products, with ease the

value of A is found by substituting in

$$A = \frac{.0r \times (1.0r)^t}{(1.0r)^{t+1} - 1} V.$$

Second, a more complicated problem: A city wishes to issue 6 per cent. bonds. It can raise A dollars annually to pay interest on the bonds and establish a sinking fund that shall yield 4 per cent. and mature the bonds in 40 years.

1st. When A is available to start sinking fund at time of issuing the bonds.

Let B be the amount of bonds to be issued.

At end of first year, $A(1.04) + A - .06B$.

At end of second year, $A(1.04)^2 + A(1.04) + A - [.06B][(1.04) + 1]$.

At end of third year, $A[(1.04)^3 + (1.04)^2 + (1.04) + 1] - [.06B][(1.04)^2 + (1.04) + 1]$.

At end of fortieth year, after multiplying and dividing by $(1.04 - 1) = .04$, we have,

$$A\,\frac{(1.04)^{41} - 1}{.04} - .06\,\frac{(1.04)^{40} - 1}{.04}\,B - B = 0.$$

To show how simply this is done, we write out

$$\begin{aligned} &\log 1.04 = .017033, \\ &\log 1.04^{40} = .681320 \therefore 1.04^{40} = 4.8010, \\ &\log 1.04^{41} = .698353 \therefore 1.04^{41} = 4.99306. \end{aligned}$$

For those afraid of logarithms it is not bad to take the square times the square to get the fourth power; the fourth by the fourth then by the second power to get the tenth; the tenth by the tenth and finally the twentieth by the twentieth; especially if you multiply only for the figures that will become significant in the result, dropping off the end figures.

From the above figures and formula, we have,

$$\begin{array}{rr} & .04\,\underline{|\,3.8010} \\ & 95.025 \\ & \underline{.06} \\ & 5.70150 \\ .04\,\underline{|\,3.99306} & \underline{1} \\ 99.8265A = & 6.7015B \end{array}$$

From this you determine either the bond issue for a given tax levy, or the tax levy A necessary to issue a given amount of bonds.

Second, in case the tax levy A is available only when the first interest on the bonds is due the formula may easily be seen to be

$$A\frac{(1.04)^{40}-1}{.04}-\left[.06\frac{(1.04)^{40}-1}{.04}+1\right]B=0.$$

When A is \$20,000 the respective values of B are \$297,922 and \$283,593.

Although these formulas are to be found in texts on investments, the reader is usually not led to see how he can adjust them to his own immediate needs, or devise other formulas. This fact and the presentation of a concise form of solution furnish excuse for this note.

The second class of problems that come, deals with the theory of probability in its native state, *i. e.*, respecting games. The question sent in "to decide a wager" was: In a game of auction pinochle, the deck containing 48 cards, two of each color from 9 to ace, three players each holding 15 cards and 3 in the blind, what chance has a player, who has neither of the two aces of hearts in his hand, of drawing one ace of hearts from the blind?

After inquiry about games, of which the writer is profoundly ignorant, it is known that the solution may be made most clear to anyone knowing the elements of the theory, as follows:

Probability of getting a hand without either ace of hearts is

$$\frac{{}_{46}C_{15}}{{}_{48}C_{15}}=\frac{33\cdot 32}{48\cdot 47}=\frac{22}{47}.$$

Probability of getting 3 in the blind including at least one ace of hearts is

$$\frac{{}_{33}C_3-{}_{31}C_3}{{}_{33}C_3}=\frac{31\cdot 31\cdot 3\cdot 2}{33\cdot 32\cdot 31}=\frac{31}{176}.$$

Probability that both these events happen is the product of

their probabilities, which is $\frac{31}{376}$.

To others it may suffice to put

$$\frac{{}_{46}P_{15}({}_{33}P_3 - {}_{31}P_3)}{{}_{48}P_{18}} = \frac{31}{376}.$$

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New York City.

THE ORDER OF TEACHING THE PARTS OF THE CALCULUS.

By John K. Lamond.

When the average student reaches the calculus his ideas of a function, a variable, a variable approaching a limit, etc., are so vague, and oftentimes so entirely wrong, that it would seem wise, since the foundations of the calculus rest so largely on the theory of limits, to spend enough time at the beginning of the course to develop the theory of limits in a careful and thorough manner.

It is the present tendency to mix the differential and integral calculus. That is, to develop the two divisions side by side. Theoretically this may seem like an ideal thing to do, for the student will be made to see the interrelation of the two divisions of the subject from the very start. But in practice, since the ideas of the calculus are so new to the student, and so very much bigger than anything which he has encountered during his previous mathematical experience, it seems doubtful if the *average student* gains anything from such a treatment. Certainly he loses nothing, if the formulas for differentiating are developed first, with no mention made of the inverse operation, and the subject is not so likely to become confusing to the poorer students.

Having developed these formulas, the thing that the student is interested in is not another series of inverse operations. The thing he is continually asking himself and his instructor is, "What is this derivative, this thing we have spent several weeks in learning how to find, really good for now that we have it?" This natural curiosity is amply satisfied by maxima and minima, and rates. Having finished these subjects, if the other courses which the student is taking are such that some knowledge of integral calculus seems desirable, the indefinite integral, or both the indefinite and definite integral, may be very profitably introduced at this time, leaving the remaining topics of the differential calculus for later treatment.

From this point it seems doubtful if one can say just what is

best. Each individual teacher, knowing the capabilities and needs of his class, is the best judge of what to omit, and what to give, and the order of the giving.

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BY C. C. GROVE.

The purpose of the committee in setting this topic is not quite clear. All that I can say, it seems, must have occurred to every one that has taught the subject. Some of the orders of presenting any school subject may be termed the historical order, the logical order, and the psychological order.

In the case of the calculus, the historical order is twofold according to the point of view or of the time and place of beginning. Despite this dilemma for us, there is much to be said in favor of an historical order of presenting a subject. It seems to fit into the student's growing capacities. He likes to see any living thing develop. In mathematics, however, it would sometimes be quite at variance with the logical order into which we so naturally go the more a subject becomes crystallized in our minds.

Last evening a friend was speaking of the piano and organ as means of musical expression. He felt the piano superior because its impulses of tone, its discontinuous notes, are suggestive and excite the imagination; whereas, the sustained notes of the organ melt into one grand finished product that we quietly admire. It seems to me that the logical order is rather like the organ, and that it is quite easy to have a presentation so logically coherent, and so naturally and simply and clearly knit together that the student feels nothing remains to be said or done, and yet becomes hopelessly lost when he tries to reproduce that "simple" presentation. The student's mind too needs to traverse some, at least, of that devious path by which the instructor reached his present logical formulation.

The third order combines the former two with the consciousness of that living interaction between instructor and class that enables the topics to be brought up at the pedagogically critical moment. This will vary somewhat with every combination of teacher and class. Thus it is, in part, that so many dif-

ferent textbooks appear. The teacher must direct this procedure carefully and good results are likely to follow.

As to the differential and the integral parts of the calculus, several years of teaching according to each of five different texts have led me to prefer to develop all the differential formulas before introducing integration, as the reverse process and then as a summation. We find no difficulty in maintaining interest through the more formal part, or any part for that matter. There is a growing interest as new power is gained to attack problems. The development of the subject is continually reviewing the mathematics studied before and bringing up new applications of that former material that furnish true delight to the student.

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New York City.

By Ross W. Marriott.

The subject of to-day's discussion is one which demands the close consideration of all teachers of the calculus, and to my mind resolves itself into the question as to whether we should hold to the time-honored custom of the presentation of the calculus or whether we should fit it to the needs of the present-day student. It seems to me, for example, that to take up the whole of the differential processes without considering the process inverse, is as unnatural as it would be in arithmetic or algebra to treat multiplication of all types of numbers before considering the operation of division, which I believe is seldom if ever done. Just as we sometimes treat division as a multiplication process, so we have occasion at times to treat integration as a process of differentiation. The processes of differentiation and its inverse are so closely allied that I believe there is an advantage gained by studying the elementary standard forms of the integration at the time we study the differential forms. Such modes of integration as require a transformation process could well be left to study under the integral calculus proper.

I think the order in which the calculus was invented has had a great deal to do with the manner in which it has been presented. When the differential calculus was invented it was found that the inverse process gave results identical with the older integral

calculus, which depended in no way upon the differential, hence the formal division into the two branches of calculus.

The demand for the calculus is continually growing, and comes from a varied class of students. Such subjects as engineering, chemistry, economics, and biology all have a claim upon calculus as an auxiliary, but many of the students of these subjects cannot afford to give time to an extended study of calculus.

If, then, we find that the calculus by a different mode of presentation better meets the needs of our students, we are justified in making such a presentation of its parts.

The arrangement of the parts of the calculus taught depends upon the object or end for which the student takes the calculus. It must be so selected that the work does not degenerate into mere mechanical routine, while at the same time the student becomes well grounded in the formal processes which are so necessary for the intelligent application of any branch of mathematics.

Simple practical application of the elementary portions should be introduced very early, the geometrical and physical significance of the derivative as soon as it is defined, and problems relating to it, may be introduced. Some of the elementary portions of curve tracing, maximum and minimum and rates can be taken up with advantage along with the formal processes of differentiation.

However, the practicality may be over done in the early stages of the calculus, and the student may lose sight of the significance of the formal processes, and so never be able to make much use of the calculus, as the real applications require a thorough notion of the formal processes.

A certain well-accepted textbook on calculus gives the symbol for an inverse circular function, and then states in italics that it is the angle whose sine is so and so. Shortly afterwards it makes an application in which this function enters additively with a pure scalar number. A student not long ago asked how it were possible to add them and get the measure of an area. This is what I call too much practicality or rather too much without sufficient preliminary formality.

I do not think such subjects as Taylor's theorem, theorem of mean, expansion of functions, etc., which are primarily applica-

tions of the differential calculus should be left until all the formal processes of integration are completed, as is done in so many textbooks, for these things have as wide an application as the calculus proper, and are indeed necessary for a logical and rigorous development of the calculus.

The significance of the calculus, then, the possibility of applying it in other fields, in short, its usefulness as an instrument should be kept constantly before the student during the study of the subject rather than deferred to some indefinite future.

A well-known educator has said that there is no more vicious educational practice, nor scarcely any more common one, than that of keeping the student in the dark as to the end and purpose of his work, for it breeds indifference and despair. The significance and usefulness of the calculus should not be kept from the student by following a time-honored custom, as the mysteries of a secret society are kept from the initiate until he has mastered the preceding degrees.

Swarthmore College,
Swarthmore, Pa.

SHOULD ARITHMETIC BE TAUGHT TO ALL PUPILS IN THE HIGH SCHOOL? WHEN? HOW MUCH TIME SHOULD BE GIVEN TO IT?

By Frank H. Scobey.

There is in my opinion no doubt whatever but that some arithmetic should be taught to all pupils in the high school. I do not know that there is any well-defined opposition to such opinion, but I do know of some who are not in favor of a review which covers the ground of arithmetic in the same way as it was done in the grammar school and with such objection to arithmetic in the high school I am entirely in sympathy.

In nearly all of our New Jersey school systems arithmetic is taught in the eight grades of the elementary school and judging from the students who come to our normal school about 25 per cent. of these have reviewed the arithmetic of the seventh and eighth grades sometime during the four years of the high-school course.

It would seem as though eight years is long enough to spend upon this beginning branch of mathematics without carrying the subject into the high school. It would be were it not that parts of the subject are beyond the mental grasp or maturity of mind of the pupil at the time they are presented. I do not think that little children of the first and second grades, as a rule, can comprehend the abstractions of number or pure number relations. These children would make more rapid progress if the study of the facts and processes of number were begun two years later. The next four years should be devoted to perfecting them in accuracy and a reasonable degree of rapidity in the fundamental processes, fractions, decimals and the elements of percentage with just enough rationalization of these processes and application to their surroundings or environment to lead them to understand and appreciate the purpose of arithmetic.

These are the years when children are most interested in mechanical processes. It is, when all things are considered, the

period during which they make the most rapid progress in calculation with reasonably large numbers.

In the seventh and eighth grades, that is those which immediately precede the high school, there should be an enrichment of the course in the way of applications. These I briefly mention under the heads of mensuration of some of the planes and solids, taking those which can be made concrete through the use of simple apparatus; applications of percentage to such business as we suppose a pupil of these grades can understand, such as profit or loss, a little of commission; taxes in connection with town expenses and such other very simple applications of arithmetic to social and industrial life as will appeal to their experience, emphasizing any application of community interest. Topics that pertain to investments of money, stocks and bonds, bank discount and exchange are often meaningless particularly when these exercises have no better bases than those afforded by the definitions and meager information of the textbook. But even where these topics are well taught I often hear my pupils say: "I never understood stocks and bonds or bank discount from my study of them in the grammar school."

The place for these topics of arithmetic is in the high school when the pupils can bring to them more maturity of mind and when they may often be correlated with topics of like nature and which belong in the high school. For illustration: If pupils take up a commercial course in the high school the study of discount should be in connection with commercial paper; stocks and bonds with the study of business associations or corporations. Just as the mathematics of the school shop, of domestic science, of agriculture are best studied where these activities are carried on. Even if these subjects are not included in the high school it is better to wait until such a time as the student has sufficient maturity of mind to understand something about the conditions upon which they are based and this is not before the high-school age. Probably the later they can be deferred in the high school the better.

Such topics as Euclid's method of highest common divisor and least common multiple of decimal numbers should be eliminated from arithmetic. Many teachers prefer to retain these on account of their supposed discipline. Whatever we may think

about this they should be taught in connection with the literal processes in algebra. If nothing more practical than a good test of the power to multiply and divide correctly they do afford that.

Cube and square roots of decimal numbers are more easily rationalized if associated with like algebraic processes. I do not think these topics have a place below the high school.

The progressions were always a part of the older arithmetic but have long been relegated to their place in algebra.

Many applications in percentage may be made a part of algebra, where the use of X for the unknown quantity facilitates or abbreviates the process. Some of you can go back with me to Olney's complete algebra, which devoted a considerable portion of the book to the topics of percentage and its applications. Some of these are obsolete now, but the plan of making algebra an instrument for generalizing the processes of arithmetic is a good one.

When a teacher in the high school some years ago, it was the custom to set aside a period of the last semester for a review of arithmetic While this is the plan of which I do not approve I refer to it merely for the purpose of remarking that the students who were preparing for college brought to it a maturity of mind and a consequent interest because of this better understanding.

Many of our high-school students enter our normal schools, where the work should be that of adapting the subject matter of arithmetic to the grades of the elementary school and studying as far as possible the method of teaching it. It is a great handicap and one of general complaint in normal schools that our pupils do not understand the subject matter of arithmetic. Time must be taken to teach the arithmetic that should have been acquired in high school.

While I believe that all pupils in the high school should be taught some arithmetic it is better if possible that its applications be made to new fields. Correlations should be made as indicated with the higher branches of mathematics, with commercial and other pre-vocational subjects, with the physical and economic sciences.

As the use of arithmetic is to make concrete or determine the

quantitative side of these subjects the time for teaching it must always be wherever the opportunity arises and in close connection with the subject itself. The amount of time devoted to it can only be determined by the nature of the subject with which it is associated and the need of the pupil but the drill in the use of figures should be just as thorough and the pupil be made as efficient as though a term were set apart for the study of arithmetic.

State Normal School,
Trenton, N. J.

By Amy L. Clapp.

Evidently, we agree as to the child's great and lamentable ignorance of arithmetic—the only question to be considered concerns the remedy to be applied. The most obvious one is "to give all pupils arithmetic during their first term in the high school." This seems hardly efficient, for, besides discouraging the pupil by repeating exactly the same subject that she has had, and often disliked, in the elementary school, we should also have to use the same methods that have been used before. Can we expect that our training, unlike that of the faithful elementary teacher, will endure permanently?

It seems to me that it would be more strategic to approach the subject from a different angle, that of algebra, and, besides gaining the increased interest due to a new subject to shape the course that it will definitely help the situation in arithmetic.

The pupil's weakness in arithmetic can be classed under two heads:

1. Lack of general mathematical common sense.
2. Inability to calculate quickly and accurately.

This first includes many things—among them is ability to read the problem and to reason. I need not try to prove to this group that algebra will help here in short word problems, that will teach the pupil to think more clearly. Then the pupil's ignorance of "short cuts" and slowness to comprehend and use them when taught can be helped if she is shown how they depend upon algebraic principles—*e. g.*, $678 \times 245 - 678 \times 145$; or 51×49. Ease in solution of percentage problems can be increased if the equation is used. Lastly, the pupil's knowledge or rather lack of knowledge of fractions can be helped if we follow the ex-

ample of those teachers who use algebra explicitly to cast light on arithmetic. In teaching algebraic fractions, they refer to the, theoretically, familiar arithmetical fractions, and lose no opportunity to review fractions by giving both numerical substitutions involving fractions and also equations to be checked that have fractional roots.

This last touches on what I think is the most serious phase of the whole situation—the inability to calculate quickly and accurately, and, what is far worse, the habit of the pupil to pride herself on the fact that she cannot, for instance, add. She seems to regard the elementary operations as childish and quite beneath the notice of a person of her advanced age. In our school, we are trying to correct this by giving to all our commercial girls a daily drill in accuracy. At present, half of these are using the Courtis Practise Tests, and the results are so good with this half that hereafter we expect to do all our drilling by means of these. This regular drill accomplishes two things, first, it increases their accuracy and speed, second, and more important, it is subtly changing their attitude towards such work. My own little beginners in algebra were really mortified the other day when they made careless mistakes in adding $+27$ and -19 —a welcome change from the high-school student's usual attitude! Next term, we expect to give to all our pupils entering from the grammar school, regardless of their course, this same drill in the Courtis Practise Tests.

So much for the question of arithmetic during the first year of the high school—the decision as to whether a girl is later to take it depends, I think, largely on what she intends to do after graduation. Some definitely need it for their future training—*e. g.*, the commercial girls must take commercial arithmetic in their junior year in connection with their bookkeeping, and those girls preparing to go to the normal school must have a half year of arithmetic during their senior. But generally, it seems as if, with this first year drill, a girl could, more profitably, spend her time elsewhere in mathematics than in arithmetic.

South Philadelphia High School for Girls,
Philadelphia, Pa.

BY RUTH MUNHALL.

To the first of these queries I would answer yes—decidedly yes. Arithmetic should be taught in the high school, and if the courses were not so crowded I should say to all pupils; for I have found almost without exception that the chief difficulty that besets the girl in algebra is an inability to perform simple arithmetical operations correctly; and that most of the failures are due to inaccuracy—and a good stiff course in arithmetic would go far to remedy this one besetting sin common to nearly all pupils.

But of course I fully realize that the time given to one subject is limited and that my desire to give all girls a good stiff course must be modified so I will take the subject up in four divisions; dealing separately with the four courses that we offer in the Philadelphia high schools.

First. The College Preparatory Course.—Here, we all realize, the work is very heavy, but it seems to me that we could slip in a little practice in old-fashioned mental arithmetic, which would be of great benefit to them all. It would help them to think more quickly and more accurately. A five-minute drill each time the class meets would be possible; in fact I am trying it in my senior class and the girls seem to enjoy it; as yet it is too early for me to say how profitable it will prove to them, but I am pretty sure that it will be worth the effort. Whether the senior year is the best place or not for this work I cannot say, but under present conditions it seems the logical place to put it.

Second. The General or Normal School Preparatory Course.—With us these girls do have a half year of arithmetic, but it seems to me that a longer course would be advisable for these girls are to be the future teachers of the children whom we will eventually get and if we could impress upon these potential teachers the importance of arithmetic we would in a roundabout way be preparing better material for the high school, as far as mathematics is concerned. This work ought to be in the senior year and ought to continue for a whole year. But just here we meet with the fact that an algebra review is given to the general girls for the first half of the senior year. And this is necessary, but would it not be possible to give these girls five hours of mathematics instead of three and thus give a longer time to the

arithmetic, either alternate the subjects or put the algebra into the first third and allow the arithmetic to take the last two thirds of the year.

Thirdly. The Domestic Science or Home Economics Course.—These girls should have a course in arithmetic during the first or second year—preferably the second. This course should comprise simple computations such as bills, budgets, household accounts of all sorts, measurements, estimation of the amount of material needed—given the dimensions, drill work for accuracy, some formula work. A very good thing would be to have a course in the senior year open to these girls. In this course more difficult work could be taken up and they, having more mature minds, would be able to better realize how much they need the work.

And last but by no means the least important comes the commercial course and there is one thing about which I have strong convictions, and that is that it should not be given in the first year. I have taught it to both freshmen and sophomore classes and I can truly say that the freshman class is not able to take in what the second year pupils can. I want a year of preparatory mathematics—algebra—then a year of good solid commercial arithmetic with plenty of drill work to try and develop accuracy. Then if the course can permit of it, it would be well to give these commercial girls a chance to have a term of arithmetic in their senior year. This course to treat of some of the harder commercial transactions. If this course is required put off teaching building association, stocks and bonds, and kindred subjects until the senior year. I have never tried this but I think it would work out well for these subjects seem hard for the majority of the girls in the first years, since they have no knowledge of business and lack this foundation, which they acquire as they take up their commercial subjects.

In looking back it seems to me that I have asked for a good deal but I am sure not for more than is needed. And I can reiterate my first statement: Yes—decidedly yes. Arithmetic ought to be taught to all pupils in the high school.

HIGH SCHOOL,
GERMANTOWN, PA.

MATHEMATICS CLUBS.

By Louisa M. Webster.

Believing that courses in mathematics are second to none in value as a mental discipline, it seems meet that the teacher's best efforts may be profitably spent in devising plans which will stimulate a desire for research. I speak from personal experience when I say that one of our most difficult problems lies in making provision for the many points of a crowded curriculum which must be treated lightly, assigned for outside work or entirely omitted.

I remember, a most forceful lesson on roulettes was given as a club paper by a student whose time was not limited. She had done considerable reading, her facts were arranged systematically, she reduced much of her reasoning to the level of the lower-class students, and treated the subject more exhaustively than could have been done during the time allotted to one recitation.

That much valuable information is crowded out of the curriculum, that many most important facts are pushed aside from classroom work, that the time allowed to lectures is far too short to satisfactorily cover the majority of points which even the average student would find interesting are well-known and much-to-be-regretted conditions. The Mathematics Club offers a remedy, and also an opportunity for considering attractive views of the subject which find no place in the classroom.

The Hunter College Mathematics Club was organized by Professor Requa "as the result of a desire on the part of both the teaching and student bodies to investigate matters connected with mathematics, to study the phases of mathematical development which are crowded out of classroom work, and to keep the students in touch with the best thoughts of the times. It aims to be a source of profitable pleasure." That it has proven so all members will testify.

The meetings are held once a month from October to June. The first of each semester is largely a social function—a wel-

come to the freshmen. About every third year we invite, as guests of honor, the graduates of the mathematics department who have won some distinction. Our alumnæ members are loyal. We seldom have a meeting that is not attended by several. Many are located in out-of-town schools. They write of their experiences, and this is helpful to the undergraduates; it shows them the practical side of the young teacher's work, the many rounds of the professional ladder, and the ever-increasing demand on the teacher's equipment. We take this as an evidence of their continued interest in the work, and their willingness to lend a helping hand.

Professor Requa says a few words at each meeting. Her talks offer suggestions for study or research or a comment on a magazine article or newspaper clipping. They always stimulate a desire for the further development of her topic. Every member of the teaching staff takes a deep interest in the work of the club. One serves as treasurer, collecting the 50 cents dues and acting as advisor for expenditures; one has general supervision, and others contribute papers and help wherever and whenever it is possible. While the topics are assigned to the students, each speaker works out her own subject matter. Usually the talks are given without notes, other than the citations of references. Illustration by models and blackboard drawings is extensively used. Inspiration and profit have also been derived from the talks given by men and women prominent in the mathematical field.

The president, vice-president, and secretary are members of the student body, chosen from the sophomore and junior grades, to serve a year. The candidates must have a record of superior scholarship and their interest in the club work must have been proven in some definite way, not merely by attendance at meetings. The president is always a junior who must have shown marked ability in her chosen field, also administrative powers. She hands the office to her successor as she reaches the upper senior grade. We consider the active co-operation of the students in the management of the club a valuable adjunct to their training for practical life. They become thoroughly impressed with the importance of study and research. Within the past three years eighteen of the graduates of the department have

taken their master's degree. Several who have done some graduate studying together presented the department with a set of books they had found specially useful, as an expression of appreciation of the benefit they have derived from the club.

Our club has been in existence eight years. It is a pleasure to report that two presidents have received high-school appointments, one is studying medicine, two have temporary assignments for college work, one is teaching in an elementary school, and another, now studying law, won the scholarship at New York University, when a member of the Woman's Law Class.

A list will show that the topics are chosen with reference to their mathematical or scientific interest.

The Parallel Axiom: Dr. F. Parthenia Lewis, Goucher College.

The Quadrature of the Circle: Dr. Elizabeth B. Cowley, Vassar College.

The Three Normals of the Parabola: Mr. John H. Denbigh, Morris High School, New York.

The Fourth Dimension: Dr. Feldman, Curtis High School, Staten Island.

The Application of Higher Mathematics to Business Principles: Dr. Schlaucht, High School of Commerce, New York.

Points on Which to Judge a Recitation in Mathematics: Dr. Breckenridge, Teachers' College.

The Benefits to be Derived from the Study of Mathematics: Dr. David L. Arnold, Julia Richman High School.

A Special Course in Geometry: Miss Matilda Auerbach, Ethical Culture School, New York City.

The Golden Age of Mathematics: Professor Emma M. Requa, Hunter College.

The Nature of Mathematical Knowledge: Professor Emma M. Requa, Hunter College.

Zero: Professor Julia Chellborg, Hunter College.

Calculating Machines: Professor Lao G. Simons, Hunter College.

Reports on Papers Read at Math. Assn. Meetings: Miss Evelyn Walker, Hunter College.

Old Mathematical Instruments: Miss Marcia Latham, Hunter High School.

Sonya Kovalesky: Miss Jean Robertson, Hunter High School.

The Geometry of Movement: Miss Martha Shott, Hunter High School.

Motion Pictures in Mathematics: Miss Grace Peters, Hunter High School.

Infinity: Miss Julia Simpson, a Hunter Alumna now on the Germantown High School staff.

Roulettes: Miss Edith Bainton, a Hunter Alumna of the Julia Richman staff.

The following have been given by student members:

The Mysticism of Numbers.
The History of Japanese Mathematics.
Baron Napier—His Life and Works.
What Women have Accomplished in Science.
Digital Reckoning.
Mathematical Fallacies.
The History of Time-pieces.
The Transit.
The Anaglyphs.

Appended List.

The Trisection of the Angle.
The Materials Used in the Teaching of Mathematics.
The Fundamental Principles of Universals.
The Russian Method of Multiplication.
The Different Proofs of the Pythagorean Formula.
The Properties of Number.
Algebraic Fallacies.
Non Euclidean Geometry.
The History of the Metric System.
Alligation.
Lincoln's Debt to Mathematics.
Magic Squares.
Ross's Blocks.
The Use of Imagination in Mathematics.
The Watch as a Compass.
The Life and Work of Galileo.

Mathematicians Who Have Become Famous in Other Fields.
Mathematics in Astronomy.
The Meaning of Billion.
Mathematics in Nature.
Number Games.

"Every Little Movement."

No longer does the college maid
Waste time or midnight oil,
Projections, trig. and logarithms
She's mastered without toil.
The higher math., all clear appears,
She sends all care away,
But integration is the thing
That holds the floor to-day—Ah!

Ev'ry little symbol has a meaning all its own,
Ev'ry integration by new formulæ are shown,
And hopeful feelings
That came a-stealing
O'er your being
Now fly despairing
As you work on, with some new methods,
Little methods, all, all your own.

It makes no difference, high or low,
Your "dates" go on the same;
When someone mentions tests are near
You work with might and main.
You take your seat in confidence,
But signs which once seemed clear
Now seem to mean a thousand things
Which cannot fit in here—Ah!

Ev'ry little symbol has a meaning all its own,
etc., etc.

L. Hertz, 1910.

O Dear! What Can the Matter Be?

O dear! what can the matter be?
Dear, dear! what can the matter be?
O dear! what can the matter be?
This problem won't work out right!
We've struggled, we've juggled, equations we've buggled;
We've added, divided, subtracted—we tried it
Most sweetly, discreetly, and then we decided
To let it severely alone.

Second Verse.

O dear! what can the matter be?
Dear, dear! what can the matter be?
O dear! what can the matter be?
Th' ghost of the problem remains.
It haunts us, it taunts us, with errors it flaunts us;
We try to efface it, but mem'ry won't chase it—
When, lo, inspiration!
With courage we face it, and quickly we make it our own.

Number Song.

Tune—"Lulu Is Our Darling Pride."

Numbers are our pride and joy,
Numbers great, numbers small;
Juggle with them like a toy
At our beck and call;
Nor does vast infinity
Puzzle or distress us;
Nor can zero (hard to see!)
In the least oppress us!
Numbers are our pride and joy,
Numbers great, numbers small;
Juggle with them like a toy
At our beck and call.

I know of no better way for a body of undergraduates to keep abreast of the times than through the medium of a well-organized club. It brings them in touch with the ever-increasing enrichment of their subject. The discussions furnish opportunities for the expression of individual ideas and for the application of theories. The importance of research and graduate work is emphasized. The members of the club show a deep interest in this phase of their education and we feel assured that most of them leave us with a feeling that they have been introduced to an extensive and interesting field of thought and labor.

Notices are given of the meetings of the several mathematical associations and the students are invited to attend. When the meetings are held in New York an official representative of Hunter College Mathematics Club attends and gives a report. Out-of-town meetings are reported by members of the teaching staff.

Hunter College,
New York City.

THE CONTENT OF A MATHEMATICAL COURSE FOR THE JUNIOR HIGH SCHOOL.*

By F. W. Gentleman.

In view of the fact that junior high schools are actually being established in different parts of New England, it becomes the duty of this association to consider what shall be the nature of the work in mathematics for the course.

The junior-high-school period, in general, comprises the seventh, eighth and ninth school years, so the outline I am to present will be for a three-year course. One of the changes to be made for junior high schools is the gradual introduction of departmental work to bridge the gap from one-teacher to many-teacher instruction. This presupposes well-prepared teachers. Another change is the unification of the work in each subject, resulting in the establishment of a more connected, more logical, system. Hence it will be possible to offer much earlier some of the less difficult and more useful of the present high-school material, and to defer some of the more difficult and less useful material now offered in the elementary schools. If we attempt to saw off a strip of the present high-school course and nail it to a strip sawed off from the present elementary-school course and to claim thereby to have made a junior-high-school course, we are surely deceiving ourselves and defeating one of the main purposes, I believe, for which the junior high schools are being established. The change offers an opportunity for very necessary reforms in the content of the course and in the method of presentation.

Instruction in mathematics in the junior high school must necessarily begin where the pupil has reached as a result of the work in arithmetic for the first six years. It is commonly conceded that during this school period (Grades I.–VI.) he should have mastered the mechanics of arithmetic, and that the six

* Read at the Springfield meeting of the Association of Teachers of Mathematics in New England, March 3, 1917.

years of school time is ample for the accomplishment of this purpose. By the mechanics of arithmetic I mean the processes of addition, subtraction, multiplication and division of integers, common fractions and decimal fractions. His attention thus far has been necessarily focused on the individual figures of numbers rather than on the number values so expressed.

In the mathematical work of the junior high school, the pupil should accustom himself to the standards of the business world; namely, that an example done once without review or check is only half done and that the responsibility for the correctness of the work must rest with the computer. The first of these means that the pupil must be shown, and compelled to use systematically, some method of reviewing or checking up his work. The second means that the responsibility for the correctness of his work must fall upon the pupil himself, and not upon some authority over him.

In the operation of addition and subtraction of numbers, no result should be accepted which does not carry with it the evidence of having been checked up. An accurate check in multiplication is performed by interchanging the multiplier and the multiplicand. An accurate check in division is performed by multiplying the quotient by the divisor and adding the remainder.

In the operation of multiplication and division, results should be estimated before any computations are performed. These estimates should be part and parcel of all the work submitted. This means that emphasis should be placed upon rational or common sense methods of locating the decimal point. Furthermore, if the method of multiplication taught were one in which the figures of the multiplier were used from left to right, then the approximation and the mechanics would go hand in hand; since the decimal point is located in the first partial product and the succeeding partial products are added as corrections thereto. This method, in later scientific work where approximate numbers are involved, lends itself to a considerable economy in the number of figures used.

In all this work, and in that which follows, process and speed should not be stressed to the sacrifice of judgment regarding results, nor to the sacrifice of the accuracy of results. Again, the choice of problems for application here and later should be

more carefully considered than appears to have been done in many current textbooks. Problems coming from impossible conditions, and naming impossible prices, are to be avoided, as well as problems that are mere collections of words to make some process necessary. Advertised sales, household budgets, statistical reports, etc., offer legitimate sources for problems. Furthermore, if we are to ground the pupil thoroughly in the fundamentals of arithmetic, so that the business world will be better satisfied with our product, we should make the wording of the problem simple and direct until the one principle involved in that problem has been mastered by the pupil. One of the main weaknesses in our problem work of to-day in the elementary course, is that many problems are so involved that the pupil becomes accustomed to failure and loses confidence in his ability to handle any arithmetical computations that require thought on his part.

Concerning the subject of percentage, from my teaching experience I am convinced that the presentation of its elements in the usual order of Cases I. and II. should be reversed. That is to say, that first the pupil should learn to interpret and to express, as per cent. relations, those relations between the number data in the fields familiar to his common experience. It is necessary for him to get a clear conception of the idea of per cent. as so many hundredths, or such a fractional part of a given amount. After the pupil has comprehended the per hundred idea and has realized that the per cent. obtained is a per cent. of some definite amount, then he may use intelligently the per centum idea in finding per cents of amounts; he is no longer in the dark as to the interpretation of the computation called for in these problems, so in making his computations he can proceed to apply common sense.

In considering the applications of percentage, it is important to note that the per centum idea has a much wider field of application than the monetary field alone. These applications should include problems concerning school data, comparisons of lines and of areas, and comparisons from the field of statistics, etc.

The third case, so-called, of percentage, the formal methods of computing interest, successive discounts, marking goods to

sell at a gain per cent. on the marked price, the computing of per cents in statistical work to certain degrees of accuracy, introductory treatment of taxes and insurance, should be deferred until the second year.

The equation should claim an early place in the course of the junior high school. It naturally occurs in the following topics: statements of elementary number-facts; formulas of mensuration, about which I shall speak at length later; statement of the equality of two ratios, here serving as a simple means of approach to the ratio idea involved in the per cent. relation; formulas for percentage, for interest and for scientific facts; and statement of general problems not included in any of the above, many of which problems have heretofore required a solution by the analytic method. The equation affords a simple direct method of expressing mathematical relations. By its use mathematical solutions are clarified. It is needed by the future mechanic and other tradesmen if they are to read trade journals intelligently, since the equation is the world's way of expressing a rule.

The kind of equation most needed by the pupil in the first and second year of the junior high school, is that which requires the axioms of multiplication and division for its solution. The treatment of the equation in these years should be natural and informal. The pupil should indicate definitely his progress, step by step, and should definitely assure himself of the correctness of his result by a check.

Another topic that should occupy an important place in the early part of the junior-high-school course is the measurement of the familiar geometrical figures and the drawing of these figures to full size and to scale. Familiarity with their shapes and properties, and a knowledge of the terms applied to them will remove many of the difficulties and misconceptions that are met later in the systematic study of geometry. For this work in measurement, the pupil should be supplied with a protractor for angle measurement, and with a ruler on which the inch is graduated to tenths as well as to sixteenths and the foot graduated to hundredths. With such a ruler measurements may be obtained in decimal fractions as well as common fractions, thereby extending the work in decimal computation beyond the monetary

field. Such a ruler would be especially useful in making scale drawings.

Special emphasis should be placed upon the care with which the measurements are made. Here may be made a study of the shape and properties of the square, rectangle, triangle, parallelogram and trapezoid. For the second year, this work should be extended to include the use of the compass for simple geometrical constructions, such as the drawing of perpendiculars, parallels, bisectors, etc. At this time the different plane figures should be systematically grouped and their common properties studied.

The graphical representation of number-data from the field of statistics has a place in the junior-high-school course. By comparing the lengths of lines and rectangles and parts of circles representing certain groups of facts, the pupil may learn to read number-data so expressed and also have experience in making graphs from given data. This work will assist him in reading intelligently the many articles appearing in magazines devoted to scientific and social problems. Furthermore, through such work he can visualize the idea of "round" number, one kind of approximate number.

The mensuration of the areas of the square, the rectangle, the parallelogram, the triangle, the trapezoid, and the circle is work within the comprehension of the first-year pupil. The rules for these should be developed from diagrams and these rules expressed as formulas at once. All this work should be on a rational basis, and a formula such as $A=bh$, should mean, first of all, A (the number of square units)$=bh$ square units. For this the unit of area (a square something) must be visualized. We should be sure that the pupil carries away no such incorrect ideas as 5 in. $\times$ 8 in. $=$ 40 sq. in. For this work in mensuration the pupil should obtain as much of his data as possible by actual measurements, then his common sense and judgment may be better trained in the use of data that he may have in any problem. If this sort of training in computation be given him in the first year of the junior high school, then in the mensuration of the second year the distinction between a number obtained by count and one obtained by measurement may be made. On this foundation, approximate computations may be taken up and

the results retained to that number of figures that corresponds to the given data of the problem.

Accuracy (so-called) where results from measured data are required to several decimal places, gives the pupil the wrong idea of what is meant by mathematical accuracy. The pupil should be trained to realize what sort of an answer he ought to get, then make an effort to get the result correct to a degree that his common sense demands. When asked your age you do not say that you are 25 years, 3 months, 5 days, 4 hours, 8 minutes and 30 seconds old. Even were such an answer correct, it would be practically senseless. When such a kind of "accuracy" is required in answers, problems cease to be real and results to be of value.

This discussion concerning the rational kind of answer leads to the topic of square root, which should come in the second year. The one really rational method for getting square root is that known by some as Newton's method. The "completing the square" method quickly degenerates into the following of a mechanical rule. For Newton's method the knowledge of many squares becomes essential, the rational estimate is all-important, and the idea that the square root is one of the two equal factors of a number can not be lost sight of.

This is the natural place in the course to apply one of the most useful facts of geometry, the Pythagorean theorem. Following this, a study should be made of the development and use of the formulas for finding the surfaces and volumes of the block, the cube, the prism, the cylinder, the pyramid, and the cone. Here, too, some of the formulas based upon the simpler facts of elementary science might be introduced to advantage, as elementary science is likely to have a place in the course of the junior high school.

Near the end of the second year, equations involving the axioms for addition and subtraction should be studied, and this followed by a study of the solution of simultaneous linear equations by the method of elimination by addition or subtraction.

Throughout the course, the teacher should require that the work be neatly done; that clear, concise statements be made showing the progress of the work, when necessary; and that the computations be systematically arranged.

At the end of the first two years of the junior high school, the pupil, who has followed the course as outlined above, should be well grounded in the fundamentals of arithmetic, so that he can attack with confidence the problems that he is likely to meet, should he be forced to leave at the end of his eighth year. He should have a fairly clear idea of the shapes and properties of the common plane figures and solids. He should have some grasp of generalized arithmetic, fitting him to continue his work in his ninth year much more intelligently than he does at present.

In the last year of his junior-high-school course, he should again make a study of arithmetic, to get some idea of its unity and its general values; he should consider more extensively the applications of percentage to the field of science, getting some definite idea of the per cent. of error in data and in result, together with a more definite knowledge of the meaning of significant figures. He should make a somewhat more intensive study than before of such applications of percentage as taxes, bonds, mortgages, insurance, etc.

Once more he should deal with formulas, but this time with the transformation of those formulas already familiar to him, and with the building of new formulas from rules, whether these rules are the results of his own experience, or not; in other words, he should now be able to symbolize his scientific language.

The addition, subtraction, multiplication and division of polynomials should be studied as an aid in solving certain types of equations. A study should now be made of linear equations, involving the treatment of negative numbers; of simultaneous linear equations, including the graphical solution; of complete quadratic equations solved by factoring; and of simultaneous equations, one quadratic and one linear, solved by substitution and by graphs. Enough factoring would be necessary to make it possible to solve any quadratic having rational roots.

Near the end of this year, I would give the pupil an introduction into the systematic study of geometry, proving informally with him certain fundamental statements, and having him prove certain other statements formally, choosing for such formal demonstrations those statements the proofs for which are clear-cut and definite.

In conclusion, it seems to me that, if the course for the junior high school is arranged somewhat as I have outlined, and certain changes of method are made as outlined, then such results as the following might be expected:

A better understanding of quantitative relations;

A more common sense viewpoint concerning the value of results, with a growing respect for sensible accuracy;

More strongly developed habits of self-reliance;

A more thorough grounding in the fundamentals of arithmetical computation;

Earlier development of the power of independent reasoning in mathematical work;

For the pupil who leaves at the end of the eighth or ninth year, a greater ability to handle a variety of the mathematical tools used in the solution of the problems of everyday life;

For those who go to the senior high school, a course better fitted to suit their capacities and tendencies, since the teacher in charge would have wider opportunity than now to see the pupil's interest in, aptitude for, and capacity to grasp the mathematical point of view;

Finally, a stimulation of his interest in the further study of mathematics by giving him a clearer idea of what the subject is about, and by presenting to him a vision of the extent of the applications of mathematics to the different fields of the world's work;

For the pupil who enters the senior high school, less danger of unwise choice of mathematical work, because he already has a fair knowledge of his mathematical limitations;

Finally, a keener interest in the further study of mathematics, because he has a clearer idea of its meaning and a vision of its manifold applications to the world's important work.

Proposed Outline.

First Year.

1. Review of fundamental operations of arithmetic.
2. Equations: (*a*) $bx = c$; (*b*) $\frac{x}{a} = \frac{b}{c}$ (ratio).
3. Measurement of straight lines, angles and plane rectilinear figures. Drawing to scale. Straight line graphs.

4. Percentage: (*a*) Finding what part one number is of another.
   (*b*) Finding percents of given amounts.
   (*c*) Applications: single discount, simple interest, commission.
5. Mensuration: (*a*) Areas of rectangular figures and the circle.
   (*b*) Rectangular and circular graphs.
6 Summary: Applied problems.

Second Year.

1. Review of fundamental operations of arithmetic applied to business transactions; approximate products.
2. Formulas: (*a*) Perimeters and areas of plane figures.
   (*b*) Square root; Pythagorean theorem.
   (*c*) Surfaces and volumes of solids.
3. Percentage: (*a*) Finding the base, percentage and rate given.
   (*b*) Applications: successive discounts, interest (formal method), notes, savings banks, taxes, insurance.
4. Construction of geometrical figures; classification of plane figures; graphical representation of statistics by broken line.
5. Equations: (*a*) $ax + b = cx + d$.
   (*b*) Simultaneous linear.

Third Year.

1. Review of arithmetic, to include bonds, mortgages, taxes, insurance, significant figures.
2. Formulas: (*a*) Transformation of formulas.
   (*b*) Construction of formulas.
3. Linear equations: (*a*) Involving addition and subtraction of polynomials.
   (*b*) Involving multiplication and division of polynomials.
   (*c*) Involving negative numbers.
   (*d*) Simultaneous (graphs).
4. Factoring: (*a*) $aQ + bQ - cQ$; (*b*) $Q^2 - O^2$; (*c*) $Q^2 + 2QO + O^2$; (*d*) $Q^2 + aQ + b$; (*e*) $aQ^2 + bQ + c$.

5. Quadratic equations: (*a*) Pure; (*b*) complete (solved by factoring); (*c*) simultaneous, one quadratic and one linear (solved by substitution and by graphs).
6. Introduction to systematic study of theorems of geometry.

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NEW BOOKS.

Rosemary. By Alice E. Allen. Boston: The Page Co. Pp. 96. 50 cents.

A good story about twins and their schooling.

Virginia of Elk Creek Valley. By Mary Ellen Chase. Boston: The Page Co. Pp. 297. $1.35 net.

This sequel to "The Girl From the Big Horn Country" is full of life, and contains many interesting characters.

A Place in the Sun. By Mrs. Henry Backus. Boston: The Page Co. Pp. 410. $1.35 net.

This is an interesting story of an animated young girl who succeeds in spite of the disadvantages of birth, by placing great faith in the theory that here in America every one has a chance to win.

The Barbarian. By Brewer Corcoran. Boston: The Page Co. Pp. 305. $1.50.

The hero of this story was an ungainly country boy who goes to a boys' school and on account of a sensitive nature makes few friends at first. He has a good mind and is gradually led out into the interests of the school and other boys. A good, wholesome story.

Analytic Geometry and Calculus. By Frederick S. Woods and Frederick H. Bailey. Boston: Ginn and Co. Pp. 527. $3.00.

The present work is a revision and abridgment of the authors' "Course in Mathematics for Students of Engineering and Applied Science." The condensation of a two-volume work into one volume has been made possible partly by the omission of some topics which can well be postponed to a later course, but largely through a rearrangement of subject matter and new methods of treatment.

The rearrangement of material is especially seen in the bringing together into the first part of the book of all methods for the graphical representation of functions of one variable both algebraic and transcendental. This has the effect of devoting the first part of the book to analytic geometry of two dimensions, that of three dimensions being treated later when it is required for the study of functions of two variables. The transition to the calculus is made early through the discussion of slope and area. From this point on the methods of analytic geometry and the calculus are intermingled.

Among the subjects omitted are determinants, much of the general theory of equations, polars, and diameters related to conics, evolutes,

complex numbers, and some types of differential equations. The book is intended primarily for first and second years in college or technical school. The number of problems offered, some two thousand in all, permits a variation of assignments from year to year.

A Course in Mathematical Analysis. By EDOUARD GOURSAT. Authorized translation by E. R. HEDRICK and OTTO DUNKEL. Boston: Ginn and Company. Volume II. Part II. Differential Equations. Pp. 300. $2.75.

Since the appearance in translation of Volume I, this treatise has exercised an increasing influence on mathematics instruction in the United States and is now recognized as one of the standard reference texts. Its wide use is due not only to the reputation of its author but to its clarity of style, its wealth of material, and the thoroughness and rigor with which the subjects are presented.

The second volume, in its French form, has long been as well and as favorably known as the first. It has now been radically revised, and the present edition is a translation of the revised edition of the text. Both volumes, in the American edition, are distinguished by unusual excellence of typographic workmanship and careful accuracy of translation.

Volume I treats the subjects studied in a second course in calculus in American colleges and prepares the way for the study of the higher branches of analysis—notably differential equations and the theory of functions. Volume II, issued in two separately bound parts, each of which corresponds to a course in many American colleges, covers the theory of functions of a complex variable and the theory of differential equations.

Laws of Physical Science. By EDWIN F. NORTHRUP. Philadelphia: J. B. Lippincott Company. Pp. 210. $2.00 net.

The general propositions or laws of science, are the fundamental basis of all exact knowledge and mastery of physical forces, and their application. What man knows of the world he lives in, is dependent upon the recognition and application of these principles. Hitherto these have not been collected in one volume for ready reference and to enable the student and reader to obtain under a single view the very epitome of the world's heritage of exact knowledge. Professor Northup has performed a valuable service in filling this obvious gap in the literature of physical science.

The student in one branch of science, who has found it difficult to gain a knowledge of the important principles and facts in other brances, will find in this book exactly what he seeks to obtain—a broad view of the entire field of natural law. To those who wish to gain an intelligent grasp of our rich mental inheritance, without having the time or means to give years of study to the search, the "Laws of Physical Science" will come as a boon and a stimulus to further investigation and wider reading. The book is divided into six sections: mechanics; hydrostatics,

hydrodynamics and capillarity; sound; heat and physical chemistry; electricity and magnetism; light. Of pocket size and weight, the volume is admirably fitted to be the student's daily companion as an indispensable book of reference.

Preliminary Mathematics. By F. E. Austin. Hanover, N. H.: the author. Pp. iv + 169. $1.20.

While this book was originally written as a help for those who wished to improve their mathematics without going to school, it has been adapted for use as an auxiliary text. The first part is designed for junior high schools, the second section for high school use.

The author connects arithmetic and algebra, giving some excellent practice in the operations of arithmetic, and gradually bringing in the use of the algebraic notation and methods.

Throughout the book, and especially in the second section, the emphasis is placed on the solution of problems. This work is rather uneven, the analysis being well done in some cases, and poorly done in others.

A Brief Account of Radio-Activity. By Francis P. Venable. Boston: D. C. Heath & Co. Pp. vi + 54.

Dr. Venable has written this book to supplement the meager treatment usually given in text books on chemistry. It covers the history of the discovery of radio-activity, the properties of the radiations, the changes taking place in the bodies, the alpha particle, a discussion of the atom, and a chapter on the connection of all these matters with the theory of chemistry.

Vocational Mathematics for Girls. By William H. Dooley. Boston: D. C. Heath & Co. Pp. vi + 369.

This is the most complete attempt that has been made to prepare girls for the use of the mathematics needed in their business relations. A foundation is first laid by a review of arithmetic, with some extensions, such as training in reading plane and drawing to scale. Over one hundred pages are then given to the problems in home making, and the rest is divided between such subjects as "Dressmaking and Millinery," "The Office and the Store," "Arithmetic for Nurses" and "Problems on the Farm." In addition there is an excellent appendix containing material of general value, or of less common application to women's work.

The book gives the impression of not only teaching arithmetic, but combining it in interesting form with much valuable knowledge on other subjects.

NOTES AND NEWS.

THE twenty-eighth meeting of the Association was held Saturday, April 28, 1917, at the junior school, Trenton, N. J. The morning session was opened at 10:45 with an address of welcome by Anna P. Hughes, vice-principal of the junior school. The topic for the morning was "Mathematics in the Junior High School." Very interesting papers on this topic were presented by the following speakers: William Betz, East High School, Rochester, N. Y.; Charles Barton Walsh, Ethical Culture School, New York City, N. Y.; Harrison E. Webb, Central High School, Newark, N. J.; Howard F. Hart, Montclair High School, Montclair, N. J.; Louise Northwood, Junior School, Trenton, N. J.

Between the morning and afternoon sessions there was an inspection of the new junior school.

In the afternoon session a very interesting paper on "Household Arts Arithmetic," written by Katharine F. Ball and Miriam E. West, of the High School, Plainfield, N. J., was read by Miss Ball.

The topic "Composite Courses in High School—Their Content, Their Strength, Their Weakness" was then discussed by the following speakers: George Alvin Snook, Frankford High School, Philadelphia; Martha W. Crow and Helen S. Opp, West Philadelphia High School for Girls; Margaret Groff, South Philadelphia High School for Girls.

The last topic of the afternoon, "Composite College Courses in Mathematics," was discussed by Floyd F. Decker, Syracuse University, Syracuse, N. Y.; William J. Fite, Columbia University, New York City.

At the close of the program Professor C. B. Upton, of Teachers College, New York City, gave a short and interesting talk on the subject of "Fusion in Mathematics in the Secondary Schools."

There were about 100 in attendance at the meeting and all present seemed to feel that it had been a very profitable meeting.

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A MAGAZINE DEVOTED TO THE
INTERESTS OF TEACHERS OF MATHEMATICS

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CONTENTS

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To solve $\frac{\partial^2\varphi}{\partial t^2} = \sqrt{1+(\Delta h)^2}\,\frac{\partial^2\varphi}{\partial x^2}$, put $m^2 = \sqrt{1+(\Delta h)^2}$ and assume $\varphi = \tau(t)\cdot\xi(x)$; so that $\frac{\partial^2\varphi}{\partial t^2} = \frac{d^2\tau}{dt^2}\cdot\xi(x)$, and $\frac{\partial^2\varphi}{\partial x^2} = \tau(t)\,\frac{d^2\xi}{dx^2}$. Substituting these into the original equation, we find that the variables, t and x, can be separated by dividing through by $\tau\cdot\xi$ whereupon we have $\frac{d^2\tau}{dt^2}\div\tau = m^2\frac{d^2\xi}{dx^2}\div\xi$. Since the first of these two equal members cannot vary when t changes nor the second when x changes both must remain equal to some constant, say $-m^2n^2$. The two resulting equations yield the solutions

$$\xi = K_1\cdot\sin[nx+\beta_1], \qquad \tau = K_2\cdot\sin[mnt+\beta_2]$$

whence $\qquad \varphi = K_1K_2\mathrm{Sin}[nx+\beta_1]\ \sin[mnt+\beta_2]$

which we may then reduce to a more useful form :

$$\varphi = \sum_{n=0}^{n=\infty} A_n \sin[n(x\pm mt)+\delta_n].$$

An interesting fallacy results from applying the method of integration by parts, $\int u\cdot dv = uv-\int v\cdot du$, to a case where $u=1/x$ and $dv=dx$: we get

$$\int\frac{dx}{x} = \frac{1}{x}\cdot\int dx - \int x\cdot[-1/x^2]$$

$$= \quad 1 \quad + \int dx/x \quad \text{whence } 0=1\ !!$$

$$\int\frac{dx}{5+7x^2} = 1/5\int\frac{dx}{1+{}^7/_5x^2} = \frac{1}{5}\sqrt{{}^5/_7}\int\frac{\sqrt{{}^7/_5}\,dx}{1+[\sqrt{{}^7/_5}\,x]^2} = \frac{1}{35}$$

arctan $[\sqrt{{}^7/_5}\ x]$.

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